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Answering a basic objection to bang/crunch holography

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Abstract: The current cosmic acceleration does *not* imply that our Universe is basically de Sitter-like: in the first part of this work we argue that, by introducing matter into *anti-de Sitter* spacetime in a natural way, one may be able to account for the acceleration just as well. However, this leads to a Big Crunch, and the euclidean versions of Bang/Crunch cosmologies have [apparently] disconnected conformal boundaries. As Maldacena and Maoz have recently stressed, this *seems* to contradict the holographic principle. In the second part we argue that this "double boundary problem" is a matter not of geometry but rather of how one chooses a conformal compactification: if one chooses to compactify in an unorthodox way, then the appearance of disconnectedness can be regarded as a *coordinate effect*. With the kind of matter we have introduced here, namely a euclidean axion, the underlying compact euclidean manifold has an unexpectedly non-trivial topology: it is in fact one of the 75 possible underlying manifolds of flat compact four-dimensional euclidean spaces.

Keywords: AdS-CFT and dS-CFT Correspondence, Cosmology of Theories beyond the SM.

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1. De Sitter ... or anti-de Sitter?

From the fact [1, 2] that our Universe is accelerating, it is often deduced that our spacetime basically resembles de Sitter spacetime. But is this justified?

There are in fact general reasons [3, 4, 5] for thinking that, despite appearances, our world is basically unlike de Sitter spacetime. For example, *holography* does not work very well for de Sitter-like spacetimes. Clearly de Sitter spacetime itself does not have a holographic euclidean version, since its standard euclidean version has no boundary [see however [6]], and its lorentzian holography [7, 8, 9] leads to problems [10] which have only been incompletely resolved [11, 12, 13].

It is therefore important to note that a Universe which accelerates only temporarily — and we have no reason to believe that it does otherwise — need not be related to de Sitter spacetime at all: in fact, *it can be modelled by a suitably modified version of anti-de Sitter spacetime.* In view of all this, it is reasonable to ask: can we dispense with de Sitter spacetime altogether, and construct a complete cosmology using only anti-de Sitter physics?

AdS_4 itself is of course completely unacceptable as a cosmological model, since it does not expand, has no Big Bang, and is not globally hyperbolic [so that it cannot be understood as a FRW cosmology — it has a FRW coordinate system, but this only covers a small part of the spacetime]. All of these blemishes are removed, however, by introducing matter into AdS_4. In fact, by introducing scalar matter into AdS_4 [14] one can produce a globally hyperbolic spacetime which does have a Bang and also a *temporary* burst of acceleration. [See also [15].] Such spacetimes are therefore ideally suited to answering questions about the global structure of spacetimes which accelerate and yet *are not asymptotically de Sitter,* because the acceleration is only temporary. [For another approach to using AdS in cosmology, see [16].]

The cosmological models obtained in this way underline a fundamental fact which deserves more attention than it has received: AdS_4 is unstable to singularity formation.

The introduction of *any* amount of *any* form of matter [other than vacuum energy itself] satisfying the Strong Energy Condition causes singularities to develop, producing a spacetime *which is not a small perturbation* of AdS_4. [This follows from the Hawking-Penrose cosmological singularity theorem [17]. See also [18] for a good recent discussion of this theorem. Note that singularities will *usually* form even if the matter does not satisfy the SEC.] Singular versions of AdS_4 are therefore the generic spacetimes with negative cosmological constants, and so it is quite natural for an AdS_4-derived spacetime to accommodate a Big Bang [and to be globally hyperbolic].

However, putting a Bang into AdS_4 has a price: it almost inevitably produces a Crunch. This is because, as the Universe expands, it dilutes all forms of matter and radiation [except phantom matter [19], which we do not consider here] more rapidly than a cosmological constant, so the latter will become increasingly important as the expansion proceeds. The negative cosmological constant of AdS_4 will inevitably halt the expansion and cause a contraction. If the Universe is still contracting when matter and radiation again become dominant, there will be a Crunch at the end just as there was a Bang at the beginning. Hence a quasi-realistic cosmological model arising from AdS_4 has *both* a Crunch and a Bang. [For the basic theory of Crunch cosmologies, see [20].] Because these AdS-derived cosmologies are singular, their global structure is radically different to that of AdS itself, even though their matter content may be a small perturbation away from the "matter content" [the negative cosmological constant] of AdS. In particular, there is no longer any timelike spatial infinity. Hence it is no trivial matter to extend the AdS holographic description [21] to them, and indeed one might think that this is not possible: perhaps holography forbids a Crunch. Recently, however, this question has been investigated [22] in the "intermediate" case where singularities are not forbidden but the boundary conditions are forced to be asymptotically AdS. The conclusion is that reasonable asymptotically AdS data do evolve to a Crunch, and that this may well be correctly represented in the CFT at infinity. Because the spacetimes considered in [22] have boundary conditions specified at spatial infinity, they are not cosmological spacetimes, but this work does show that holography, and in particular string/M theory, is *not* fundamentally incompatible with a Crunch. This prompts the question: how is it possible to obtain a holographic description of a Crunch in the context of a genuine cosmological model?

Maldacena and Maoz [23] have recently argued that it might be possible to establish such a description of AdS-derived Bang/Crunch cosmologies by focusing primarily on the *euclidean* domain. In fact, the euclidean versions of Bang/Crunch cosmologies are automatically non-singular and otherwise [apparently] very well-behaved. Thus while, for example, a lorentzian Crunch cannot be associated with a critical point of a superpotential, it may well be possible to achieve such a description of the non-singular object which replaces the Bang and the Crunch, that is, the euclidean conformal boundary. *The Maldacena-Maoz proposal is that singular Bang/Crunch cosmological spacetimes are described by a dual field theory defined on the non-singular boundary of the euclidean version.* Thus, the euclidean version of a Bang/Crunch cosmology is taken to be the fundamental version, the properties of the lorentzian spacetime being derived from it.

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Now the euclidean manifolds considered by Maldacena and Maoz apparently have conformal compactification with boundaries *having two disconnected components*. Thus, the problem of obtaining a holographic understanding of cosmology seems to involve extending the [euclidean] AdS/CFT correspondence to spaces with multiple boundaries. This raises two issues, one mathematical, the other physical.

The *mathematical* issue is that it is not always *possible* to have two boundaries when physically reasonable conditions are imposed on the geometries of the bulk and the boundary, a point first made in [24]. The subtle way in which the Maldacena-Maoz cosmologies [including, especially, versions with temporary acceleration] evade the Witten-Yau theorem was the subject of [25], and we shall not consider this question further here: we merely note that it is *not* in fact possible to prohibit multiple boundaries in this way.

The *physical* issue is that having two [or more] boundary components for one bulk seems to be a blatant contradiction [24] of the holographic philosophy, which is founded on a putative one-to-one bulk/boundary correspondence. *In general* this is a very deep question: some relevant ideas are advanced in [23] [see also [26]] and, in the lower-dimensional context, in [27] and [28]. We wish to argue, however, that in the *cosmological* context [though not in general] this is something of a red herring. What is special about the cosmological context is that the two boundary components have to have the *same topology*, since topology "change" in the euclidean version would entail topology change in the course of the evolution of the lorentzian version, and it is well known that this leads to various difficulties [29, 30] under physically reasonable conditions. This means that we should ask whether the "two" boundary components *might perhaps be one and the same*. Obviously this would solve the problem in the simplest and most direct manner.

We shall present a concrete example of a quasi-realistic anti-de Sitter based cosmology. We begin, motivated by the remarks in [4], by formulating a euclidean superpotential for an axion field, and show briefly how to adjust the parameters by using observational data. We then show that there is a way of interpreting the euclidean metric such that conformal infinity is clearly connected, despite the existence of both a Bang and a Crunch in the lorentzian spacetime. If we accept this [mildly] unorthodox way of assigning a conformal infinity to a non-compact euclidean manifold, then the double boundary problem simply arises from a bad choice of coordinates; for the conformal compactification is *not* a manifold-with-boundary but rather simply a compact manifold, one of the possible underlying manifolds of flat compact spaces in four dimensions.

It turns out that there are several possible topological structures for this manifold, just as, in the more familiar two-dimensional case, a flat compact manifold can be either a torus or a Klein bottle. We argue that the choice should be made on physical grounds, taking into account the fact that the matter here is axionic. The result is that the euclidean conformal compactification is topologically non-trivial. Thus the real significance of the double boundary problem is revealed: it is simply telling us that the euclidean version has a more intricate topology than at first appears. *This structure is, however, fully compatible with the holographic principle.*

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2. A euclidean axion

As we have explained, the euclidean spacetimes in which we are interested are topologically non-trivial in the sense that [in the most obvious interpretation of the metric] the boundary is disconnected; Maldacena and Maoz describe them as [euclidean] "wormholes", though of course the lorentzian interpretation is not a wormhole in the usual sense. Thus the matter we shall introduce into euclidean AdS_4 has to be carefully chosen; most forms of matter do not lead to topologically non-trivial spaces. Maldacena and Maoz [23] use meron and instanton Yang-Mills configurations, which automatically satisfy the Strong Energy Condition and hence cannot lead to even temporary cosmic acceleration. In order to improve on this we need a kind of matter which can lead to

[a] temporary acceleration in the lorentzian version and

[b] topological non-triviality in the euclidean version.

In view of point [a], the natural kind of matter to consider is scalar [or perhaps pseudoscalar] matter, since it is well known [31] that such matter ["quintessence"] can lead to acceleration. However, it is also known that scalar fields cannot lead to topologically non-trivial asymptotically flat euclidean configurations, as in point [b], unless the field itself is complexified upon passing between the lorentzian and euclidean domains. For this the reader may consult [32]; see also the comments in [33] and [34]. All these works are directly relevant only to the asymptotically *flat* case; but it can be shown that the same requirement for complexification holds in the asymptotically anti-de Sitter case also. [There are technical complications: it is no longer true, as in the asymptotically flat case, that topologically non-trivial configurations are only possible when the euclidean scalar field makes a negative contribution to the Ricci curvature; however, there are other conditions, replacing this one, which still require field complexification. See [25] for the details.]

Complexification of the field arises most naturally for *axions*. Thus points [a] and [b] above lead us naturally to the assumption that the euclidean version of the Universe is basically like euclidean AdS_4, but differs from it because of the presence of a euclidean axion, that is, a pseudo-scalar with a periodic potential. [For the use of *lorentzian* axions in cosmology, see for example [35, 36, 37].]

In [4], Hellerman et al show that permanently accelerating quintessence spacetimes do not evolve towards a supersymmetric state described by a critical point of a superpotential, and this is adduced as further evidence for the incompatibility of de Sitter-like spacetimes with string theory. In anti-de Sitter cosmology, where the universe ends in a Crunch, we have even less reason to believe that the system is evolving towards a supersymmetric state. However, in the context of the Maldacena-Maoz approach, in which the euclidean version is fundamental, we can work towards answering this objection by ensuring that the asymptotically AdS regions of the *euclidean* version do correspond to critical points of a superpotential. Since we are dealing with a euclidean axion, which should have a periodic potential, we can express the superpotential $\mathrm{W}^+(\varphi^+)$, for a single euclidean field φ^+, as a Fourier series. [To avoid confusion, a superscript + will be used where necessary to emphasise that a quantity is "euclidean", with the lorentzian version being denoted by a minus sign.] We then have

$$\mathrm{W}^+(\varphi^+) = \sum_{k=1}^{\infty} \mathrm{C}_k \sin\left(k\sqrt{\frac{4\pi}{\varpi}}\,\varphi^+\right) + \frac{1}{2}\mathrm{B}_0 + \sum_{k=1}^{\infty} \mathrm{B}_k \cos\left(k\sqrt{\frac{4\pi}{\varpi}}\,\varphi^+\right). \tag{2.1}$$

Here ϖ is a positive constant; the way in which it appears is for later convenience.

Since the sign of a pseudoscalar field has a physical significance, it is particularly interesting to consider *odd* superpotentials, so we restrict to

$$\mathrm{W}^+(\varphi^+) = \sum_{k=1}^{\infty} \mathrm{C}_k \sin\left(k\sqrt{\frac{4\pi}{\varpi}}\,\varphi^+\right). \tag{2.2}$$

It will turn out that the higher frequency modes of this expansion are important only extremely near to the Bang and the Crunch. For simplicity we postpone their consideration and consider [for now] only a single term in this expansion, namely the first:

$$\mathrm{W}^+(\varphi^+) = \mathrm{C}\sin\left(\sqrt{\frac{4\pi}{\varpi}}\,\varphi^+\right); \tag{2.3}$$

The effect of replacing this superpotential by the k-th higher-order term is of course obtained simply by replacing ϖ by ϖ/k^2.

Because the axion is complexified when passing between the euclidean and lorentzian domains, the relation between potential and superpotential here is given [in four dimensions] by

$$\mathrm{V}^+(\varphi^+) = -8(\mathrm{W}^+)'^{\,2} - 96\pi(\mathrm{W}^+)^2\,. \tag{2.4}$$

For pure euclidean AdS_4 with all sectional curvatures equal to $-1/\mathrm{L}^2$, obtained when $(\mathrm{W}^+)'$ vanishes, the potential should have its usual value for a negative cosmological constant, that is, $-3/(8\pi\mathrm{L}^2)$. This fixes the constant C in (2.3) at the value $1/(16\pi\mathrm{L})$, so we in fact have

$$\mathrm{W}^+(\varphi^+) = \frac{1}{16\pi\mathrm{L}}\sin\left(\sqrt{\frac{4\pi}{\varpi}}\,\varphi^+\right). \tag{2.5}$$

Substituting this into (2.4) we obtain, after some simple manipulations,

$$\mathrm{V}^+(\varphi^+) = -\frac{3}{8\pi\mathrm{L}^2} + \mathrm{V}^+_{\mathrm{Axion}}\,, \tag{2.6}$$

where

$$\mathrm{V}^+_{\mathrm{Axion}} = \frac{3-\varpi^{-1}}{8\pi\mathrm{L}^2}\cos^2\left(\sqrt{\frac{4\pi}{\varpi}}\,\varphi^+\right). \tag{2.7}$$

Clearly this system *appears* to be euclidean AdS_4, with "energy" density $-3/(8\pi\mathrm{L}^2)$, into which we have introduced a matter field with a potential $\mathrm{V}^+_{\mathrm{Axion}}$. Notice however that the AdS_4 energy density does not really arise independently of the energy density of the axionic matter being introduced — both have a common origin in the original superpotential.

The matter potential $\mathrm{V}^{+}_{\mathrm{Axion}}$ will be positive provided that

$$\varpi > \frac{1}{3}\,; \tag{2.8}$$

this will always be satisfied in the sequel since it will turn out that the physically interesting values of ϖ are larger than unity. Thus the matter we are "introducing" into AdS_4 is not very exotic.

The combined Einstein-axion field equations for this potential can be solved exactly if we assume that the geometry takes the euclidean FRW form with flat "spatial" sections. To be specific, we assume that the sections are compact and flat; in the simplest case they are tori, so that the metric takes the form

$$g^{+} = (dt^{+})^{2} + A^{2}\, a(t^{+})^{2}[d\theta_1^2 + d\theta_2^2 + d\theta_3^2]\,, \tag{2.9}$$

where A measures the circumferences of the torus when $\mathrm{a(t^{+})}$, the scale factor, is equal to unity.

There are several motivations for taking the "spatial" sections to be compact. In the generalized [euclidean] AdS/CFT correspondence, it is best if the CFT is defined on a compact space, so as to avoid problems with non-unique correlation functions — see section 2.3 of [38]. Note that, even in the lorentzian version of AdS/CFT, the CFT is really defined not on Minkowski space but rather [38, 39] on its conformal *compactification* or on the latter's universal cover, both of which have compact spatial sections of topology S^3. We take all this as strong evidence that cosmological holography works best — or perhaps only — for cosmologies with compact "spatial" sections in both the euclidean and lorentzian versions. Again, there are in fact very interesting string cosmological models in which it is essential that the sections should be both flat and compact: see [40, 41] and also, for a quite different model, [42].

Another motivation for taking the "spatial" sections to be compact is the following. The observational data [2] suggest that the spatial sections of our Universe are close to being flat. [The "spatial" sections of the euclidean version of the manifold have the same geometry and topology as the spatial sections of the lorentzian version.] Naturally we should be aware that the data may be misleading us: they are equally compatible with a small positive curvature. However, it is interesting to consider the possibility that the data are hinting that the spatial sections of our world *really are* flat, by which we mean that they could have a topological structure which *forbids* them to have completely positive or completely negative curvatures. This is a property [43] shared by all of the ten different *compact* three-manifolds [44] which can be flat, but not by $\mathbb{R}^3$ [which is the underlying topology of both flat and hyperbolic space in the simply connected cases]. Thus the data pointing to "exact" flatness suggest that the spatial sections are modelled on one of these ten manifolds. We have chosen the torus T^3 only for reasons of simplicity: the other nine possibilities will not be discussed here.

The combined euclidean Einstein-matter equations for the potential given in (2.6) and (2.7) yield

$$(\dot{\varphi^{+}})^{2} = \frac{1}{4\pi\varpi \mathrm{L}^{2}} \cos^{2}\left(\sqrt{\frac{4\pi}{\varpi}}\,\varphi^{+}\right). \tag{2.10}$$

This provides us with a useful consistency check: since the right hand side of this equation obviously does not change sign whether or not φ^+ is complexified when passing to the lorentzian domain, this equation only makes sense if φ^+ *is* complexified [since otherwise the left side would change sign when t is complexified and the right side would not]. Thus indeed φ^+ must be complexified, in the manner of an axion, when transforming to the lorentzian version.

The solution of (2.10) is given by

$$\varphi^+ = \pm\sqrt{\frac{\varpi}{4\pi}}\cos^{-1}\left(\mathrm{sech}\left(\frac{t^+}{\varpi \mathrm{L}}\right)\right), \tag{2.11}$$

where t^+ runs from $-\infty$ to $+\infty$, where the sign agrees with that of t^+, and where $\cos^{-1}$ is defined to take its values between 0 and $\pi/2$. This solution will be important below.

It is shown in [25] that the Einstein equation can be solved exactly here: the solution for the metric takes the surprisingly simple form

$$g^+(\varpi, A) = (dt^+)^2 + A^2\cosh^{2\varpi}\left(\frac{t^+}{\varpi \mathrm{L}}\right)[d\theta_1^2 + d\theta_2^2 + d\theta_3^2]\,. \tag{2.12}$$

This is [as foreseen] completely non-singular, with asymptotically hyperbolic regions near $\mathrm{t}^+ = \pm\infty$; that is, the metric increasingly resembles that of hyperbolic space, the euclidean version of AdS_4. This can be seen explicitly, since this metric has the approximate asymptotic form

$$g^+(\varpi, A) \approx (dt^+)^2 + 4^{-\varpi}A^2\exp\left(\frac{2|t^+|}{\mathrm{L}}\right)[d\theta_1^2 + d\theta_2^2 + d\theta_3^2]\,, \tag{2.13}$$

which is a well-known local form of the standard metric of constant curvature $-1/\mathrm{L}^2$. We see that the length scale here is just L, the curvature scale set by the asymptotic cosmological constant $-3/(8\pi \mathrm{L}^2)$. [There is no other natural length scale, because there is none associated with the flat "spatial" directions, and the range of t is infinite.]

The metric $g^+(\varpi, A)$ in equation (2.12) can be regarded, in the usual way, as a metric on the interior of a manifold-with-boundary, the boundary consisting of *two* copies of some compact flat three-dimensional manifold [such as a flat torus]. From equations (2.5) and (2.11) we see that these flat three-manifolds correspond to the critical points of the superpotential. If euclidean holography is to make sense at all in Bang/Crunch cosmology, it must apply to some kind of euclidean field theory defined on these flat spaces. A priori there seems to be no reason why this should not work. Before discussing this further, we turn to the question of the plausibility of the lorentzian version as a cosmological model.

3. Cosmic acceleration in anti-de Sitter physics

The lorentzian version of this space cannot be expected to share any of the agreeable properties of $g^+(\varpi, A)$: the boundary will be singular, there will be no superpotential with critical points, and so on. The metric is of course

$$g^-(\varpi, A) = -(dt^-)^2 + A^2\cos^{2\varpi}\left(\frac{t^-}{\varpi \mathrm{L}}\right)[d\theta_1^2 + d\theta_2^2 + d\theta_3^2]\,. \tag{3.1}$$

It can be shown that this is, as it appears to be, singular: there is a Big Bang at $\mathrm{t}^{-} = -\pi\varpi\mathrm{L}/2$, and a Big Crunch at $+\pi\varpi\mathrm{L}/2$. By complexifying φ^{+} in the euclidean potential given by equations (2.6) and (2.7), we obtain a lorentzian potential which reveals that the lorentzian field φ^{-} is a kind of quintessence:

$$\mathrm{V}^{-}(\varphi^{-}) = -\frac{3}{8\pi\mathrm{L}^2} + \mathrm{V}^{-}_{\mathrm{Quintessence}}, \tag{3.2}$$

where

$$\mathrm{V}^{-}_{\mathrm{Quintessence}} = \frac{3 - \varpi^{-1}}{8\pi\mathrm{L}^2}\cosh^2\left(\sqrt{\frac{4\pi}{\varpi}}\,\varphi^{-}\right). \tag{3.3}$$

Such potentials have been extensively explored: similar ones were investigated recently in [45] and [in the context of "racetrack inflation"] in [46]. Of course, the model we are considering here differs from these because, in equation (3.2), we are superimposing the "quintessence" potential on the negative "potential" [cosmological constant] of AdS_4. [For other exponential-like quintessence potentials see [31, 47, 48, 49, 50]; for other metrics similar to $g^{-}(\varpi, A)$, see [51].]

It is also interesting to compare this model with the one assumed in the "cyclic" scenario [52]. As in that case, the total potential here can be negative; in our case, this is a simple consequence of having a negative cosmological constant in the background. In both cases we have a Bang and a Crunch. However, there is an important dissimilarity also: we shall see that, in our case, the negative potential [which reveals itself through an equation-of-state parameter exceeding unity] is only important near to the transition from expansion to contraction, that is, well away from the Bang and the Crunch. In the cyclic scenario, by contrast, it is very important that the equation-of-state parameter should exceed unity *near* to the Bang and to the Crunch, because this is the way in which the cyclic model avoids having large Belinsky-Khalatnikov-Lifschitz anisotropies which might propagate through the "bounce" [53]. In our model there is no bounce. Nevertheless our studies of cosmic holography will lead us to a geometric structure which may be highly relevant to the "cyclic" model, as we shall discuss below.

One obtains φ^{-} in the lorentzian case either by solving the field equation or directly by complexifying φ^{+}: the result in either case is given by

$$\varphi^{-} = \pm\sqrt{\frac{\varpi}{4\pi}}\cosh^{-1}\left(\sec\left(\frac{t^{-}}{\varpi\mathrm{L}}\right)\right), \tag{3.4}$$

where the sign follows that of t^{-}. Unlike φ^{+}, φ^{-} diverges towards the [singular] boundaries of the lorentzian spacetime, as one would expect.

The energy density of φ^{-} can be computed in terms of the scale function; the result [25] is

$$\rho(\varphi^{-}) = \frac{3}{8\pi\mathrm{L}^2}\,a^{-2/\varpi}. \tag{3.5}$$

The total energy density is the sum of this and the energy density of the background AdS_4. Notice that if we had taken the k-th order term in (2.2) instead of the first, then the density of φ^{-} would vary as $a^{-2k^2/\varpi}$. If t_r is some early time, say during the era of

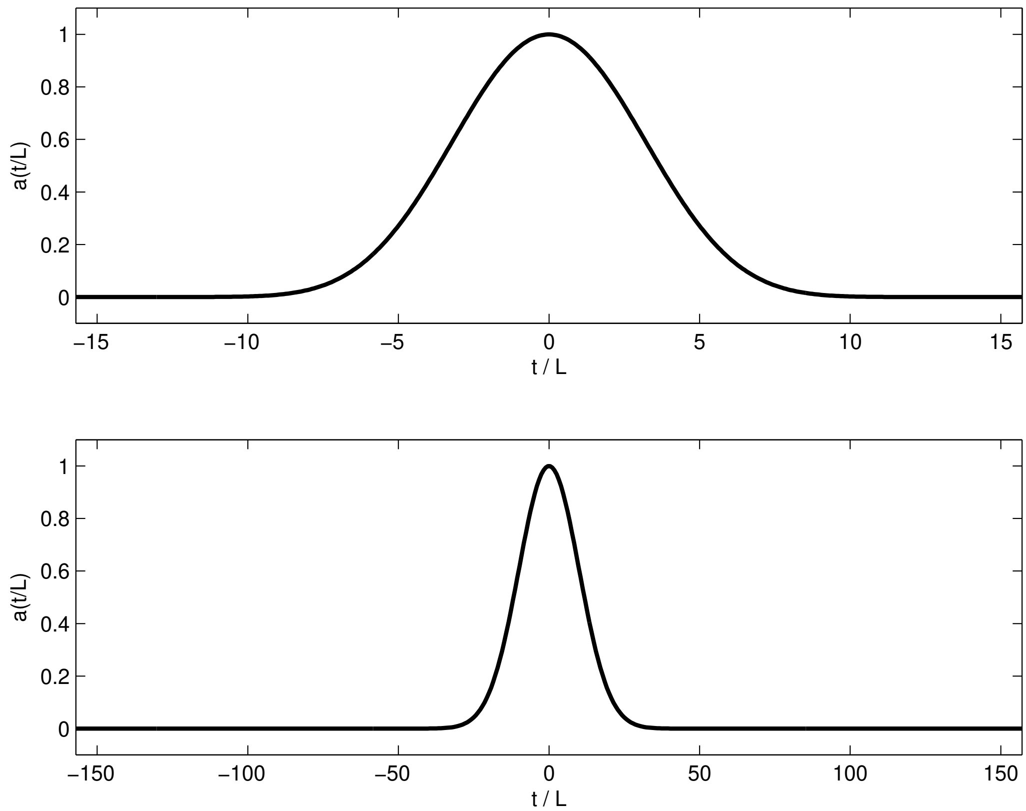

Figure 1: Scale Factor for $\varpi = 10$ [above] and 100 [below].

radiation dominance [when all components of dark energy were of negligible importance], and ρ_r and a_r are the values of the density and the scale function then, we have in the general case

$$\frac{\rho\left(\varphi^{-}\right)}{\rho_r} = \left(\frac{a}{a_r}\right)^{-2\,k^2/\varpi}. \tag{3.6}$$

Clearly the higher terms in the Fourier expansion (2.2) make a contribution which dies away more rapidly than the lower terms as the Universe expands, justifying our emphasis on the first term. [If such terms were included, they would be important at extremely *early* times, but it would not be consistent to do this since we are not including ordinary matter and radiation in our model.]

Graphs of the lorentzian scale function, for two indicated values of ϖ, are shown in figure 1. It is clear from a glance at these diagrams that *these cosmologies can accelerate*; they do so at times corresponding to those parts of the graphs which are "concave up". Note that $\mathrm{t}^{-} = 0$ is the moment of time symmetry, beyond which the Universe begins to contract: so that the present time corresponds to some *negative* value of t^{-}.

The natural length scale here is not set by the asymptotic curvature, as it is in the euclidean case. Instead, the total duration of the spacetime, $\pi\varpi$L, sets the scale of this system. Notice that this length scale need *not* be the same as that of the euclidean version, which, as we saw, is simply given by L. Thus the constant ϖ measures the ratio of the lorentzian to the euclidean length scales. In particular we can consider the possibility that L is quite small; this is compatible with a large observed Universe provided that ϖ is large.

In the euclidean version, the essential length scale reveals itself in the AdS-like asymptotic regions [see equation (2.13)], but this scale is concealed in the lorentzian version by the fact that there are no asymptotic regions in this sense — the spacetime is cut off by the Bang and the Crunch. This is of interest because the values for L associated with holographic theories can be small by cosmological standards. For example, in an AdS/CFT scheme, one expects L to be related to the coupling in the dual theory; large L will correspond to strong coupling, small L to weak. Again, it has recently been emphasised [54] that in many string compactifications, and specifically in the Freund-Rubin case [55], the requirement that the Kaluza-Klein modes be sufficiently massive as to escape observation apparently requires the fundamental length scale of four-dimensional spacetime to be far smaller than what we observe. This discrepancy may be accounted for [54] by quantum corrections, by supersymmetry breaking classical corrections, and so on; in any case, there must be some parameter, computable from these effects, which removes the discrepancy between the fundamental and the observed length scales. We can regard ϖ as a simple concrete example of such a parameter.

As we shall see, observational data do in fact require ϖ to be fairly large [for any L], and so it is natural [though not compulsory] to assume that L is small by cosmological standards; so it is possible that one can work towards resolving these issues in the way being advocated here: that is, the length scale of four-dimensional space *is* small, but only in the euclidean version. This would presumably require that the coupling in the putative dual theory is weak.

The equation-of-state parameter w which gives the ratio of pressure to density is *not* a constant, being given instead by

$$\mathrm{w} = -1 + \frac{2}{3\varpi}\mathrm{cosec}^2\left(\frac{t}{\varpi \mathrm{L}}\right)\,; \tag{3.7}$$

this is bounded below by [and comes arbitrarily close to] its value "at" the Bang,

$$\mathrm{w}_{\mathrm{Bang}} = -1 + \frac{2}{3\varpi}\,. \tag{3.8}$$

As is well known, acceleration corresponds to $\mathrm{w} < -1/3$, so *temporary acceleration will occur in this model* provided that ϖ exceeds unity, which is of course consistent with the above discussion. In that case, if we define a time t^* by

$$\mathrm{t}^* = -\varpi \mathrm{L}\sin^{-1}\sqrt{\frac{1}{\varpi}}\,, \tag{3.9}$$

then the spacetime accelerates from the Bang until $\mathrm{t}^- = \mathrm{t}^*$, it then decelerates from $\mathrm{t}^- = \mathrm{t}^*$ until $\mathrm{t}^- = |\mathrm{t}^*|$ [having started to contract at $\mathrm{t} = 0$], and then accelerates [while contracting] from $\mathrm{t}^- = |\mathrm{t}^*|$ until the Crunch. The present time corresponds to some negative value of t^- less than $\mathrm{t}^- = \mathrm{t}^*$. Notice that equation (3.9) shows that, if ϖ is large, $|\mathrm{t}^*|$ is given approximately by $\sqrt{\varpi}\,\mathrm{L}$, which is small as a fraction of the total lifetime $\pi\varpi\mathrm{L}$. Thus the interval of deceleration is relatively short in this case [see figure 1].

We observe from that, unlike some pure quintessence spacetimes in which w tends to -1, the w function here is bounded away from -1. To reconcile this with the observations, which show that w is currently rather close to -1, ϖ has to be large. To be more precise, w is currently [2] no larger than about $-\ 0.8$ [and it may be far smaller], and since the cosec function is bounded below by unity, we see from (3.7) that ϖ has to be at least 3 or 4; in fact this is a gross underestimate. Notice that this argument does *not* depend on any assumptions about the value of L; that is, ϖ has to be large [that is, significantly larger than it needs to be in order to allow acceleration] in any case. Let us be a little more precise about this.

If the present time is t_0 — recall that this is a negative number — then define α to be the ratio of $|t_0|$ to the cosmic length scale ϖL. Then α is an angle lying between 0 and $\pi/2$. The values of the Hubble constant and of w at the present time can be fairly strongly constrained by observations. Much less stringent but nevertheless non-trivial constraints can be placed [2, 56] on the rate at which w changes as we look back into cosmic history. [See however [57].] Following [56, 58], we measure this by means of the parameter

$$\mathrm{w}_a = -a(\mathrm{t}_0)\frac{\mathrm{dw}}{\mathrm{d}a}(a(\mathrm{t}_0))\,, \tag{3.10}$$

so that negative w_a means that w is currently increasing. [This parametrization is a significant improvement over the one used in [25], because it does not refer to the age of the Universe, a datum which cannot be coherently included in this model since it is so strongly affected by the matter content of the early Universe.]

The observed quantities H_0, w_0, and w_a are given in this cosmology by

$$\mathrm{H}_0 = \frac{1}{\mathrm{L}}\tan(\alpha)\,, \tag{3.11}$$

$$\mathrm{w}_0 = -1 + \frac{2}{3\varpi}\mathrm{cosec}^2(\alpha)\,, \tag{3.12}$$

$$\mathrm{w}_a = -\frac{4\cos^2(\alpha)}{3\varpi^2\sin^4(\alpha)}\,. \tag{3.13}$$

Notice that the theory predicts that w must be increasing at the present time.

Solving these three equations for the three unknowns L, ϖ, and α, we have

$$\mathrm{L} = \frac{\sqrt{\kappa^{-1}\ -\ 1}}{\mathrm{H}_0}\,, \tag{3.14}$$

$$\varpi = \frac{2}{3[1+\mathrm{w}_0][1-\kappa]}\,, \tag{3.15}$$

$$\alpha = \cos^{-1}\sqrt{\kappa}\,, \tag{3.16}$$

where

$$\kappa = \frac{|\mathrm{w}_a|}{3[1+\mathrm{w}_0]^2}\,. \tag{3.17}$$

The current value of the scale function is given by

$$a(\mathrm{t}_0) = \kappa^{\varpi/2}\,. \tag{3.18}$$

Notice that equations (3.16) and (3.17) imply that

$$|\mathrm{w}_a| < 3[1+\mathrm{w}_0]^2 \,. \tag{3.19}$$

Unfortunately the current uncertainties in the data for the quantities H_0, w_0, and w_a still allow many possibilities. To give a concrete example, let us adopt "Hubble units" in which the currently measured value of H_0 is unity. For the sake of argument we assume that w_0 is about $-\ 0.8$; this may in fact be too large: it is compatible with [2] but is questionable in view of [59]. According to the inequality (3.19), the theory itself predicts that $|\mathrm{w}_a|$ can be no larger than 12 percent. Any value of this magnitude is compatible with all observations [2, 56, 59] [though again the reader should consult [57]; see also [60]].

If we take $|\mathrm{w}_a|$ to be 8 percent, then L is 0.707 in Hubble units, ϖ is 10, so that we are in the situation portrayed in the upper panel of figure 1. The current value of t_0/L is $-\ \varpi\alpha = -\ 6.16$, and that of the scale function is [by (3.18)] about 0.132. If we take $|\mathrm{w}_a|$ to be 11.6 percent, then L drops to 0.186 Hubble units, ϖ increases to 100, and we are in the lower panel of figure 1. The current values of t_0/L and of the scale function are $-\ 18.4$ and 0.184 respectively. Examining these points on the graphs, we see that in both cases the present time corresponds to a point close to the left side of the base of the "hill". However, this is not a "cosmic coincidence": it arises because we are, for illustrative purposes, deliberately choosing extreme values of w_0 and $|\mathrm{w}_a|$. In fact, of course, w_0 could easily be much closer to -1 than the value we are discussing, and $|\mathrm{w}_a|$ could be much closer to zero; this is indeed suggested by the latest data analysis [59]. That would push the present time farther to the left on the diagrams.

If L is very small in Hubble units, a situation which, as we have discussed, may arise naturally in Freund-Rubin compactifications and from the point of view of the dual boundary theory, then (3.11) and (3.12) give

$$\varpi \approx \frac{2}{3[1+\mathrm{w}_0]}\mathrm{L}^{-2}\,, \tag{3.20}$$

so that the growth of ϖ over-compensates for the smallness of L: that is, large values of ϖ may be typical from the holographic point of view.

Our model does not incorporate any kind of matter or radiation other than φ^-, so it is pointless to attempt to give a more detailed numerical account of the observed data in terms of this model. Broadly speaking, however, the inclusion of matter and radiation would of course change the shape of the graphs in figure 1 at very early [and very late] times. In fact, moving towards the left, the graphs should turn down at a time corresponding to the observed [2] onset of acceleration. This will cut off the long, flat regions of the graphs. [It turns out that the inclusion of higher modes in equation (2.2) also has this effect; we hope to return to a discussion of this point elsewhere.]

The inequality (3.19) means that the theory requires w to change very slowly if, as is in fact the case, w_0 is close to -1. In turn, values of w_0 close to -1 are suggested by the fact that the theory favours large values of ϖ. That is, the theory itself requires that observations should continue to point towards values of w_0 close to -1 and towards small values of w_a. Contrary to what is often said, then, *observational findings of that*

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kind — see [59] — *in no way confirm the idea that the dark energy corresponds to a [positive] cosmological constant*, since it is now clear that they can be naturally described in this very different way. As figure 1 shows [see also figures 2 and 3 below], the global structure of this temporarily accelerating universe is very different indeed from that of any de Sitter-like model. This strongly underlines the fact, stressed in many recent data analyses [58, 61, 62, 63, 64], that observations showing that w varies slowly and is currently close to -1 are far from proving the existence of a positive cosmological constant.

We claim, then, that anti-de Sitter physics may be able to account for the cosmological observations just as well as its much less well-understood de Sitter counterpart. Of course, one would have to develop a realistic model containing matter and radiation to give a detailed account, but it is reasonable to hope that at least the main virtues of our "toy" spacetime will survive. This already represents progress, since there is hope of linking [the euclidean version of] anti-de Sitter physics to string theory. However, as the price to be paid is a Big Crunch, one may wonder whether this approach is still susceptible to the more general criticisms of accelerating universes given in, for example, [4].

A well-known unpleasant feature of de Sitter-based cosmologies is the "cosmic coincidence" problem mentioned earlier. This problem is best formulated as follows. In a spacetime containing nothing but ordinary matter and radiation together with a positive cosmological constant, the Universe lasts for an infinite time if it does not re-collapse before accelerating. In that case, the graph of the scale function has precisely *one* point of inflection, and there is an infinite interval of time in which, granted that we observe acceleration, we might expect to find ourselves. It is therefore remarkable that in fact we find ourselves very close [2] to the point of inflection. In our cosmology this problem is greatly alleviated. According to the present model, the cosmic acceleration will soon, by cosmological standards, come to an end, and therefore our current location in time is very much less remarkable than it would be in a de Sitter-like spacetime. This is a welcome alternative to less palatable solutions of the problem, such as anthropic ones.

More specific criticisms of de Sitter-like cosmologies focus on their causal structure. There are two main criticisms of this kind, and it is important that they be kept separate [even though they have a common mathematical origin]. The first criticism is that, in de Sitter spacetime, it is possible to find many pairs of points such that the future null cones of those points never intersect. Correlations between such events are therefore not measurable, and it is difficult to reconcile this with the existence of an S-matrix. Precisely this same observation applies to the Big Crunch of the traditional "closed" FRW cosmologies, for the same reason: the future singularity is spacelike, just as is the "infinitely inflated future" of de Sitter spacetime. However, the physical situation is very different in the case of a Big Crunch. In de Sitter spacetime, the future null cones fail to intersect even though they correspond to events on worldlines of objects which continue to exist for an arbitrarily long period of time. In a Big Crunch, by contrast, the inability of two observers to communicate beyond a certain time has a very simple explanation: they are about to be destroyed, or the Universe is about to become opaque, and so on. The failure of communication in such a situation is not a mystery and surely does not in itself indicate a failure of the S-matrix formalism.

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The second criticism is quite different, though it too has its origin in the spacelike nature of future infinity. In de Sitter spacetime each inertial observer is surrounded by a horizon, demarcating the region of spacetime beyond which no event can ever influence that observer, and this horizon apparently has physical [thermal] properties despite being observer-dependent. This leads to many complications [65] and apparent paradoxes which it would be best to avoid. Observers in Big Crunch cosmologies also are [typically] surrounded by such horizons, so it is important to investigate the parallel argument in our case.

To see what is happening here, let us temporarily allow the "maximal radius" A of the spatial tori in equation (3.1) to tend to infinity. [We do this only to clarify the subsequent argument: we continue to insist that cosmic holography probably requires compact spatial sections, as we discussed earlier.] Now it is easy to see that the lorentzian spacetime with metric (3.1) is conformal to at least part of Minkowski spacetime. To see which part, define a parameter λ by $\cos(\mathrm{t}^-/\varpi \mathrm{L}) = \mathrm{sech}(\lambda)$, so that the extent of conformal time from the Bang until the point of maximum expansion $[\mathrm{t}^- = \lambda = 0]$ is given by

$$\varpi \mathrm{L} \int_{-\infty}^{0} \cosh^{\varpi - 1}(\lambda) d\lambda \,. \tag{3.21}$$

This diverges precisely in the case of physical interest, that is, when the spacetime admits a non-zero period of acceleration: for we saw when discussing equation (3.8) that $\varpi > 1$ is the condition for this. Similarly the Crunch is at positive infinity in conformal time if the Universe accelerates. Thus, in the physically interesting case, the spacetime is conformal to *all* of Minkowski spacetime, and so the Penrose diagram is as in figure 2. [See the Penrose diagram for Minkowski spacetime, page 123 of [17].] The jagged lines represent singularities at future and past null infinity; future and past timelike infinity are likewise singular, but spatial infinity is of course non-singular.

Evidently there do *not* exist pairs of points in this spacetime with non-intersecting future null cones; there is always some observer who can correlate the two points. Furthermore, *every* event in this spacetime is visible to a sufficiently long-lived observer: there is no horizon of the de Sitter kind. Note that all this holds *provided* that $\varpi > 1$, that is, provided that the Universe has a period of acceleration. Thus the objections to de Sitter spacetime discussed above can be avoided here, and, ironically, it is precisely the presence of a period of acceleration which allows us to avoid them. The point is that a Big Crunch need not have a similar conformal structure to the infinitely inflated future of de Sitter spacetime, and hence need not have the same objectionable features.

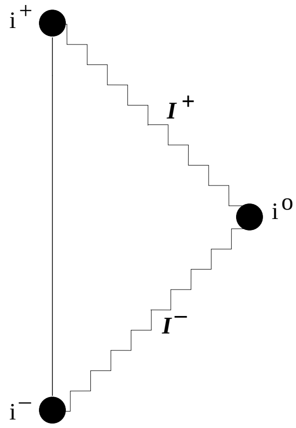

Figure 2: Penrose diagram, $\mathbb{R}^3$ Spatial sections.

If we re-impose a compact [toral] structure for the spatial sections, the effect is to abolish the null and spatial conformal infinities. Past and future timelike infinity are then just points, indicated by the large dots at the top and bottom of figure 3. To understand figure 3, the reader can imagine that the interior of a spatial torus is an infinite set of nested cubes, each cube being represented by the two-sphere in

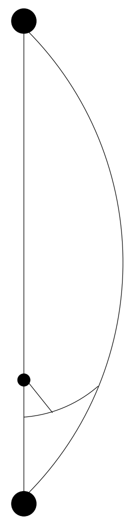

Figure 3: Penrose diagram, T^3 Spatial sections.

which it can be inscribed. There is a limiting sphere of this kind, and figure 3 is obtained by deleting all larger two-spheres from figure 2. The precise shape of the diagram will of course depend on A, the maximal "radius" of the spatial torus. Thus the figure is not entirely accurate, because more should be deleted in certain directions, but it conveys the important information here. [This spacetime, like all spacetimes with flat but topologically non-trivial spatial sections, is not globally isotropic, so it cannot be fully represented on a conventional Penrose diagram.] Note that careful study of figure 3 reveals the fact that the spacetime is in fact *null*, though not of course timelike, geodesically complete.

In figure 3, as in figure 2, there are no horizons of the de Sitter kind. This is an attractive way of avoiding the various objections which have been raised against de Sitter-like cosmologies. The curved "horizontal" line near the bottom of figure 3 corresponds to the moment when the Universe became transparent; the small dot represents the present time; its past lightcone is also indicated, under the assumption that we are not *yet* able to see an entire spatial section. This is based on the pessimistic/conservative assumption that there is no evidence of topological non-triviality in the present cosmic background radiation.

This last assumption is in fact somewhat controversial [66, 67]; we are taking this conservative viewpoint here only because we wish to stress that the compactness of the spatial sections is of fundamental importance in cosmological holography, *whether or not the corresponding non-trivial topology is presently observable.* Apart from the technical advantages mentioned earlier, spatial compactness will surely reveal itself in the dual theory, for the following simple reason: the bulk metric does not induce a metric at infinity, *only a conformal structure.* The question as to whether the spatial sections of the Universe are too large for their finiteness to be directly observable is therefore entirely irrelevant in the dual theory, where the concept of size is undefined. Indeed, if we can establish a holographic duality of the kind envisioned by Maldacena and Maoz, it should be capable of answering all questions regarding the topology of the spatial sections. Note from figure 3 that the spatial sections will *eventually* be completely visible according to our model.

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In this section we have discussed an extremely simple example in which the introduction of matter into AdS_4 produces a spacetime which naturally reproduces the observed acceleration of the Universe, and which may in fact improve on de Sitter spacetime in important ways. Obviously the model is far too simple to be regarded as the basis of a completely realistic cosmology; and on the theoretical side, too, much remains to be done. The next step would be to study in detail the dual field theory at infinity, the existence of which is the basic hypothesis of holography. Before we can hope to do that, however, there is a more fundamental issue which must be settled. We must answer the objection that, since the euclidean versions of Maldacena-Maoz cosmologies apparently reside in the interior of manifolds-with-boundary having disconnected boundaries [see for example equation (2.12)], these spacetimes actually *contradict* the holographic principle [24]. We now turn to this question.

4. Are there really *two* boundaries?

It seems obvious from equation (2.12) that the "infinity" of the euclidean version of our spacetime is a space with two connected components, one each at $\mathrm{t} = \pm\infty$. However, *this could be a coordinate effect*: recall that it also "seems obvious" that the Schwarzschild and de Sitter metrics are singular at the respective horizons if one uses certain coordinates. Note that care is needed here, because "infinity" is made meaningful in these constructions by means of a conformal transformation which "makes infinity finite", and the structure of the resulting finite space is not always obvious.

The following example should make the point clear. Consider a four-dimensional flat cubic torus with all circumferences equal to 4L and with angular coordinates θ, θ_1, θ_2, and θ_3; here the angular coordinates all run from $-\pi$ to $+\pi$. The metric is

$$g^+(\mathrm{Torus}) = \frac{4\mathrm{L}^2}{\pi^2}[d\theta^2 + d\theta_1^2 + d\theta_2^2 + d\theta_3^2]\,. \tag{4.1}$$

Now suppose that we conformally re-scale this metric along the θ direction [only], in the following way:

$$g^+\left(2, \frac{2\mathrm{L}}{\pi}\right) = \frac{4\mathrm{L}^2}{\pi^2}\left[1 - \frac{\theta^2}{\pi^2}\right]^{-2}[d\theta^2 + d\theta_1^2 + d\theta_2^2 + d\theta_3^2]\,; \tag{4.2}$$

the reason for the notation will soon become apparent. The four-torus is deformed: as we approach the three-torus labelled $\theta = \pm\pi$ from either side, the three-tori labelled by θ are enlarged, the single torus at $\pm\pi$ being "infinitely large", and also infinitely far from every other point in the four-torus. Thus the metric is singular at that three-torus, which means that the metric is really defined on the space obtained by excising this three-torus from the four-torus.

Going in the reverse direction, presented with a space with the metric in (4.2), one can strip away the singular factor, add in a three-torus at $\theta = \pm\pi$, and obtain the four-torus with metric (4.1) as the "conformal compactification" of the space with metric (4.2). Of course, T^4 is not a conformal compactification in the usual sense, since it is not a manifold-with-boundary. Nevertheless it does all that can be expected of a "conformal compactification", since it is compact and it allows us to treat infinity as a "place". This "place" is a distinguished submanifold instead of a boundary.

It is of course possible to interpret the space on which (4.2) is defined in a more orthodox way. We can do this by changing the coordinates. Define a new coordinate t^+, with dimensions of length, by

$$\mathrm{t}^{+} = 2\mathrm{L}\tanh^{-1}\left(\frac{\theta}{\pi}\right). \tag{4.3}$$

Clearly t^+ runs from $-\infty$ to $+\infty$. Now transforming the coordinate θ in equation (4.2) accordingly, we find that $g^+(2, \frac{2\mathrm{L}}{\pi})$ is given by

$$g^{+}\left(2, \frac{2\mathrm{L}}{\pi}\right) = (dt^{+})^2 + \frac{4\mathrm{L}^2}{\pi^2}\cosh^4\left(\frac{t^+}{2\mathrm{L}}\right)\left[d\theta_1^2 + d\theta_2^2 + d\theta_3^2\right]. \tag{4.4}$$

But this is nothing but a special case [$\varpi = 2$, $\mathrm{A} = 2\mathrm{L}/\pi$] of our metric $g^+(\varpi, A)$ given in the general case by equation (2.12). We repeat: equations (4.2) and (4.4) represent *exactly the same metric*, expressed in different coordinates. And yet we agreed that the "conformal compactification" for (4.2) was a four-torus, with infinity corresponding to *one* three-torus at $\theta = \pm\pi$, while (4.4) is now seen to be a special case of a family of manifolds which "obviously" had *two* three-tori as the infinity of their conformal compactifications. What is happening here?

The answer is simple. The basic idea of a Penrose compactification is that it is often possible to render singular metrics non-singular by means of a conformal transformation.[1] The resulting metric is then defined on a topologically open space. This space can then be regarded as a subspace of a *compact* space. It is not generally appreciated, however, that *there is no unique way of performing this last step.* For example, a space with topology $\mathbb{R}^2$ can be regarded as the interior of a closed two-dimensional disc; one does this when constructing the conformal compactification of hyperbolic space H^2 [which has $\mathbb{R}^2$ topology]. On the other hand, ordinary stereographic projection allows us to represent $\mathbb{R}^2$ as a subspace of the sphere S^2. Similarly, an open cylinder $(0,\,1) \times \mathrm{S}^1$ can be represented *either* as a subspace of the compact closed cylinder $[0,\,1] \times \mathrm{S}^1$ *or* as a subspace of the torus T^2, by deleting a circle from the latter. This is precisely the kind of ambiguity we are finding here.

In the analysis of singularities in classical general relativity, this ambiguity is not important because the conformally related spacetime is unphysical — it is a mathematical device. In the context of holography, however, one has to be more careful, because there is assumed to be a physical relationship between the various parts of the compactified space. We must choose our method of conformally compactifying in the physically most reasonable way. Clearly, in the cosmological context, that means that we should do it in such a way that infinity is connected. Let us see how this works in general.

[1]In fact, it is worth noting that this is *always* possible in the euclidean case: see [68].

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We shall change the coordinate t^+ in equation (2.12) as follows. Define a constant c_ϖ by

$$\mathrm{c}_\varpi = \frac{\varpi}{\pi}\int_0^\infty \mathrm{sech}^\varpi(\zeta)d\zeta\,. \tag{4.5}$$

Now define a new coordinate θ by

$$\mathrm{c}_\varpi \mathrm{L} \mathrm{d}\theta = \pm\mathrm{sech}^\varpi\left(\frac{t^+}{\varpi \mathrm{L}}\right) dt^+\,, \tag{4.6}$$

where the sign is chosen as + when t^+ is positive, $-$ when t^+ is negative. The range of θ is just $-\pi$ to $+\pi$, corresponding to t^+ ranging from $-\infty$ to $+\infty$; that is, the asymptotic regions are at the finite θ values $\pm\pi$, and $\theta = 0$ corresponds to $\mathrm{t}^+ = 0$. Now solve for t^+ in terms of θ and use this to express $\mathrm{sech}^\varpi(\frac{\mathrm{t}^+}{\varpi \mathrm{L}})$ in terms of θ. Denote this function by $\mathrm{G}_\varpi(\theta)$; then $\mathrm{G}_\varpi(\theta)$ vanishes at $\pm\pi$, and $g^+(\varpi, A)$ is given in terms of the coordinate θ as

$$g^+(\varpi, A) = \mathrm{c}_\varpi^2 \mathrm{L}^2 \mathrm{G}_\varpi^{-2}(\theta)\left[d\theta^2 + \left(\frac{A}{\mathrm{c}_\varpi \mathrm{L}}\right)^2 (d\theta_1^2 + d\theta_2^2 + d\theta_3^2)\right]. \tag{4.7}$$

This way of expressing the metric is canonical in the sense that it reveals the fact that this space is *globally* conformally flat [that is, it has not just the local conformal geometry but also the *topology* of a flat manifold, a much stronger property than the mere vanishing of the Weyl tensor]. Here we are thinking of the "spatial" sections as cubic tori, as usual. The number $\mathrm{A}/(\mathrm{c}_\varpi \mathrm{L})$ will be discussed below, but for the moment we shall set it equal to unity. Then (4.7) takes the particularly simple form

$$g^+(\varpi, A) = \mathrm{c}_\varpi^2 \mathrm{L}^2 \mathrm{G}_\varpi(\theta)^{-2}[d\theta^2 + d\theta_1^2 + d\theta_2^2 + d\theta_3^2]\,. \tag{4.8}$$

The conformal factor diverges at $\theta = \pm\pi$, so this represents "infinity" here. But does $\theta = \pm\pi$ represent one region or two? We now know the answer: removing the conformal factor, we see at once that the structure of the underlying space is most naturally interpreted as that of a four-dimensional euclidean cubic torus. "Infinity" is just *one* three-torus at *one* point on the circle parametrized by θ. Here infinity is represented not by a boundary component but rather by a special submanifold of the underlying four-torus. There is no physical reason for insisting that infinity is a boundary; it is possible to adapt the usual bulk-infinity correspondence [38] to the submanifold interpretation. The bulk is the open submanifold of T^4 obtained by deleting the T^3 at infinity.

In summary, then, the claim that "infinity is disconnected" for the space with metric (2.12) is only valid if we insist that infinity *must* be interpreted mathematically as a boundary. If we allow a slightly less orthodox interpretation, such as we propose here, then this is no longer the case. From this new point of view, the fact that infinity appears to be disconnected in (2.12) is just due to a bad choice of coordinates. Note that we are *not* of course claiming that the number of connected components of the boundary of a manifold-with-boundary can be changed by means of a change of coordinates: what we are claiming is that infinity need not be interpreted in terms of *boundaries* at all. This is in fact a variant of a known trick in topology — one is always free to interpret a compact manifold-with-boundary as a subspace of a compact *manifold*, often called its "double", and it is often advantageous to do so.

Because our space is globally conformally flat, it makes sense to speak of the shape [though not of the *size*] of the underlying four-torus. This shape is of physical interest, for it can be probed by conformally invariant physical fields, such as the Yang-Mills fields considered by Maldacena and Maoz [23]. One might suspect that it has some thermodynamic interpretation, as is usually the case when euclidean "time" is cyclic. Furthermore, if the θ direction is enormously larger than the others, then one could object that the bulk on the two sides of "infinity" is effectively disconnected for physical purposes, and then we would be trading one failure of holography — two infinities for one bulk — for another: "two" bulks for one infinity. Fortunately this is a question which can be settled by an appeal to the observational data.

The shape of the conformal four-torus is determined by the parameter $\mathrm{A}/(\mathrm{c}_{\varpi}\mathrm{L})$, which in turn is constrained by the data as follows. Let us assume that the current radius of the spatial three-torus is K Hubble distances, where K is at least unity [69], though of course it could be far larger. According to equation (3.18) we therefore have in Hubble units

$$\mathrm{A} \approx \mathrm{K}\kappa^{-\varpi/2}\,. \tag{4.9}$$

If for simplicity we pick ϖ to be the even integer 2n, then one can show by solving a recursion relation that

$$\mathrm{c}_{2n} = \frac{2^{2n}(n!)^2}{\pi\ (2n)!}\,. \tag{4.10}$$

Thus for the situation portrayed in the upper panel of figure 1, we have $\mathrm{A} \approx 7.59\mathrm{K}$, $\mathrm{c}_{10} \approx 1.29$, and since L is 0.707 in that case, we see that $\mathrm{A}/(\mathrm{c}_{\varpi}\mathrm{L}) \approx 8.32\mathrm{K}$. In the situation portrayed in the lower panel of figure 1, $\mathrm{A}/(\mathrm{c}_{\varpi}\mathrm{L})$ is only a little smaller, $\mathrm{A}/(\mathrm{c}_{\varpi}\mathrm{L}) \approx 7.32\mathrm{K}$. For large values of ϖ, Stirling's formula $\mathrm{n}! \approx \sqrt{2\pi \mathrm{n}}\ \mathrm{n}^{\mathrm{n}}\ \mathrm{e}^{-\ \mathrm{n}}$ gives $\mathrm{c}_{\varpi} \approx \sqrt{\varpi/2\pi}$ and then we find

$$\frac{\mathrm{A}}{\mathrm{c}_{\varpi}\mathrm{L}} \approx \mathrm{K}\sqrt{3\pi(1+\mathrm{w}_0)}\,\kappa^{\left(\frac{1}{2}-[3(1+w_0)(1-\kappa)]^{-1}\right)}\,. \tag{4.11}$$

This function approaches a minimum value when w_0 is taken as large as possible [compatible with observations] and w_a is taken as large as possible [compatible with (3.19)]. If, as usual, we take w_0 to be -0.8, then this value is about 3.86K, so $\mathrm{A}/(\mathrm{c}_{\varpi}\mathrm{L})$ cannot be less than this. Thus the underlying conformal four-torus is far from being elongated in the θ direction: it is in fact shorter in that direction than in the other three directions, by a factor between about 4 and 8, even in the most optimistic case in which the current radius of the spatial torus is about one Hubble distance. Thus we certainly have one bulk for one infinity.

We have seen that it is not possible to *prove* that equation (2.12) represents a space which *must* be regarded as the interior of a manifold-with-boundary having two boundary components. For we have also represented the space as an open submanifold of a compact manifold with no boundary at all; infinity then corresponds to a *connected* submanifold of that compact manifold. This is not to say that the question as to whether infinity is connected is meaningless. Rather, it means that the question is one which has to be settled by physics, not merely by inspecting the form of the metric tensor. [The same remark applies to the structure of the "spatial" sections: we have assumed for simplicity that they are tori rather than one of the other nine compact flat three-manifolds, but this is not imposed on us by the form of the metric — it is a physical question to be settled by physical arguments.] We shall now address this question.

We saw that the requirements that the lorentzian version of our spacetime should accelerate, and that the euclidean version should be topologically non-trivial, led us directly to the euclidean axion as the most natural candidate for the matter content of our model. Now combining equations (2.10), (2.11), and (4.6), we find that θ and φ^+ are related by

$$\mathrm{c}_\varpi \mathrm{d}\theta = \sqrt{4\pi\varpi}\cos^{\varpi-1}\left(\sqrt{\frac{4\pi}{\varpi}}\varphi^+\right)\mathrm{d}\varphi^+ ; \qquad (4.12)$$

for example, if $\varpi = 2$, the relation is just $\theta = \pi \sin(\sqrt{2\pi}\ \varphi^+)$. This means that we can interpret φ^+ as a *coordinate* which could be used to replace θ. As θ ranges from $-\pi$ to $+\pi$, φ^+ ranges from $(-\sqrt{\frac{\varpi}{4\pi}} \times \frac{\pi}{2})$ to $(+\sqrt{\frac{\varpi}{4\pi}} \times \frac{\pi}{2})$. These are the values of φ^+ corresponding to the two asymptotic regions. The question as to whether there are really two boundaries [that is, whether θ is really an angular variable] should therefore be formulated as: are there physical reasons for interpreting φ^+ as an angular coordinate?

Since φ^+ is an axion, the answer may seem obvious. But there is an interesting subtlety here: let us ask again: do the two extreme values of φ^+ correspond to two different physical situations?

If one begins with equation (2.7), which can be interpreted as the result of introducing a certain kind of matter [with potential $\mathrm{V}^+_{\mathrm{Axion}}$] into euclidean AdS_4, then it is indeed natural to claim that the two situations are identical. For the function $\mathrm{V}^+_{\mathrm{Axion}}$ is *periodic with period* $\sqrt{\frac{\varpi}{4\pi}}\times\pi$, so the φ^+ values $(-\sqrt{\frac{\varpi}{4\pi}}\times\frac{\pi}{2})$ and $(+\sqrt{\frac{\varpi}{4\pi}}\times\frac{\pi}{2})$ should not be distinguished. From this point of view, then, it is clear that the euclidean metric given by (2.12) is defined on the open submanifold of the four-torus given by deleting a three-torus. The conformal compactification is the full four-torus, T^4, with its metric visible in (4.7) after removing the conformal factor; *here all of the coordinates are angular coordinates.* More generally, since we do not know the topology of the spatial sections of our universe, the topology of the compactification is $\mathrm{S}^1 \times (\mathrm{T}^3/\mathrm{F})$, where F can be *any* of the entries in the known list of finite groups [44] which can be holonomy groups of compact three-dimensional flat riemannian manifolds.

On the other hand, one may feel that the superpotential $\mathrm{W}^+(\varphi^+)$ is more fundamental than the potential derived from it. This is in fact implicit in the procedure we have followed in this paper, where we are taking the euclidean point of view to be fundamental, and in which we began by postulating a euclidean superpotential given by equation (2.5). Notice that while the superpotential always occurs quadratically in the formula for the potential, it does not always do so when couplings to spinors are considered [70]; therefore we cannot ignore its sign. In view of this, it seems now that the two ends of the range for φ^+ are *not* identical, since

$$\mathrm{W}^+\left(-\sqrt{\frac{\varpi}{4\pi}}\times\frac{\pi}{2}\right) = -\mathrm{W}^+\left(+\sqrt{\frac{\varpi}{4\pi}}\times\frac{\pi}{2}\right). \qquad (4.13)$$

Thus the superpotential detects the difference even though the potential cannot. *Notice that the periodic identification we seek is not an automatic consequence of the fact that we began with a [euclidean] axion.* Indeed, it begins to seem that an axionic superpotential may actually forbid a cyclic interpretation of euclidean " conformal time".

In fact, however, this problem arises from taking a too simplistic view of the process of "identifying the ends". We have stressed that our metric allows for the possibility that the topology of the "spatial" sections of the euclidean version of the space [which is identical to the topology of the true spatial sections in the lorentzian version] could be any one of the 10 possible topologies [44] for flat compact three-manifolds. One can think of this in the following way: when constructing a torus, one identifies opposite sides of a parallelopiped, but it is also possible to do this after performing "twists" of varying degrees of complexity. This can of course also be done in *four* dimensions. There are 75 distinct possible topologies for compact four-dimensional flat manifolds, most of which are non-orientable: there are 48 non-orientable spaces and 27 orientable ones. Thus, in performing the identification suggested so strongly by equations (4.7) and (4.8), we are free to perform such a twist.

In particular, we can define a compact flat four-dimensional manifold as follows. Let a_μ, where $\mu = 1$ through 4, be an orthogonal basis for $\mathbb{R}^4$, where we take a_1 to be of unit length, while a_2, a_3, and a_4 are of length $\mathrm{A}/\mathrm{c}_\varpi \mathrm{L}$. For any real β let $\beta\,\mathrm{T}_\mu$ denote the isometry of $\mathbb{R}^4$ [with its standard metric] defined by translation by $\beta\,\mathrm{a}_\mu$. Let B be the isometry of $\mathbb{R}^4$ defined by the linear extension of

$$\mathrm{B}:\ a_1 \ \rightarrow\ a_1\,, \qquad \mathrm{B}:\ a_2 \ \rightarrow\ -a_2\,, \qquad \mathrm{B}:\ a_3 \ \rightarrow\ -a_3\,, \qquad \mathrm{B}:\ a_4 \ \rightarrow\ -a_4\,, \tag{4.14}$$

and consider the $\mathbb{R}^4$ isometry defined by $\alpha = (\mathrm{B}, \frac{1}{2}\,\mathrm{T}_1)$, meaning B followed by $\frac{1}{2}\,\mathrm{T}_1$. Clearly $\alpha^2 = \mathrm{T}_1$, and so if $\alpha\,\zeta = \zeta$ for some ζ in $\mathbb{R}^4$ then ζ is also a fixed point of T_1, which is impossible. In fact the non-abelian infinite group Γ generated by α and the T_μ acts properly discontinuously and freely on $\mathbb{R}^4$, so $\mathbb{R}^4/\Gamma$ is a smooth manifold. To be more precise, Γ is a group with no element of finite order [other than the identity] and with a maximal free abelian subgroup of index 2 and rank 4 [generated by the T_μ]. By the relevant version of the Bieberbach theorems [[44], page 105] it follows that $\mathbb{R}^4/\Gamma$ is a compact flat four-dimensional manifold. The metric is the one visible in equation (4.7) after the removal of the conformal factor. As a riemannian manifold, $\mathbb{R}^4/\Gamma$ can be expressed as $\mathrm{T}^4/\mathbb{Z}_2$, where T^4 is the rectangular oblate torus with aspect ratio given by $\mathrm{A}/(\mathrm{c}_\varpi \mathrm{L})$ as discussed earlier, and where $\mathbb{Z}_2$ is the linear holonomy group of this space. [This expression is not unique, but this will not matter here.] This allows us to think of the manifold as being like a torus which has been cut through and then glued together after reflecting one end. In fact we can regard $\mathrm{KB}^4 = \mathbb{R}^4/\Gamma$ as a four-dimensional version of a Klein bottle.

If an orthonormal basis is parallel transported around this space along the a_1 direction, it will return to the initial point with orientation reversed in the way defined by B, because B essentially generates the $\mathbb{Z}_2$ holonomy group of KB^4. The axion field φ^+ is able to detect this reversal. The upshot is that the superpotential $\mathrm{W}^+(\varphi^+)$ is no longer able to distinguish the "two" infinities, and once again we have a physical justification for regarding the "two" as one location, artificially split by a choice of coordinates. Notice that while $\mathrm{KB}^4 = \mathrm{T}^4/\mathbb{Z}_2$ is non-orientable, the reversals of orientation occur only in the "time" direction: the spatial sections [and the infinity submanifold] themselves are still orientable, being in fact copies of the three-torus T^3. *Both* bulk and infinity are orientable separately — only the combined space is non-orientable. Hence there is no conflict with the putative holographic duality, and no complications arising in connection with formulating spinors on non-orientable manifolds.

Thus we see that, whether one takes the potential or the superpotential to be fundamental, it is possible to identify the "two" boundary components which are *apparently* required by Maldacena-Maoz cosmologies, that is, by the euclidean formulation of a Bang/Crunch cosmology. If, following Witten and Yau [24], one regards a double boundary as a contradiction of the holographic principle, then surely this is the simplest possible resolution of this problem.

If we accept that this is the correct response to the "double boundary problem", then it is natural to ask what [if anything] this identification implies for the lorentzian versions of these spacetimes. The obvious assumption is that it implies that [in a sense to be clarified] *conformal* time is cyclic in the lorentzian case also. However, it is not at all clear that this is *necessary*: after all, the essence of the Maldacena-Maoz proposal is that holography works in the euclidean domain, not in the lorentzian: see figure 3. It therefore seems quite plausible to us that the lorentzian Bang and Crunch need *not* be identified. However, let us briefly explore the alternative possibility. Once again, if we are to argue that the two components of lorentzian conformal infinity should be identified, this must be done on the basis of a physical argument and not because of a formal analogy.

In the particular case of the spacetime we have been discussing here, there actually *is* such an argument: it runs as follows. Equations (2.11) and (3.4) can be written as

$$\cos\left(\sqrt{\frac{4\pi}{\varpi}}\,\varphi^+\right) = \mathrm{sech}\left(\frac{t^+}{\varpi \mathrm{L}}\right), \tag{4.15}$$

$$\cos\left(\frac{t^-}{\varpi \mathrm{L}}\right) = \mathrm{sech}\left(\sqrt{\frac{4\pi}{\varpi}}\,\varphi^-\right). \tag{4.16}$$

This clearly establishes a natural one-to-one map between the euclidean and lorentzian versions of this spacetime: leaving aside the constant factors, we can map a given t^+ slice of the euclidean space to the lorentzian slice which has t^- given by the value of φ^+ corresponding to t^+. In that sense, the euclidean axion field φ^+ *is actually lorentzian time.* Similarly, of course, the lorentzian axion *is* euclidean "time". But we have argued that φ^+ must be considered to be essentially periodic, meaning that the euclidean conformal "time" coordinate θ is an angular variable on the underlying manifold of the euclidean space. Hence we conclude that, in this spacetime, the timelike direction is periodic *after* infinity has been "made finite": that is, lorentzian conformal time is periodic. There are no closed timelike worldlines in the spacetime itself, but there *are* such worldlines in the Penrose diagram; in other words, the large dots at the top and bottom of figure 3 must be identified. This is the physical interpretation of the "twist" [reflection] that occurs as the four-dimensional Klein bottle KB^4 is circumnavigated: the twist essentially "re-sets the lorentzian clock".

The status of closed timelike worldlines in fundamental theories such as string/M theory is a subject of much current interest; see [71] for an extensive list of references to early work and [72, 73] for more recent references. In the spacetime we are considering here, such worldlines appear only very indirectly, in the spacetime with the conformally transformed metric: we stress that there are no closed timelike worldlines in the physical spacetime itself. However, the closed timelike worldlines in the conformally related spacetime may not be completely hidden, since the "unphysical" spacetime can be probed by conformally invariant fields such as gauge fields or curvature coupled scalars. It is therefore reassuring that there is no evidence in these works suggesting that string/M theory forbids such worldlines entirely. It would be of great interest to see how the discussions in [72] and [73] can be adapted to the case where the Crunch is just the Bang approached from the "other side".

The identification of the Crunch with the Bang leads to no difficulties in spacetimes as simple as the one considered here, but it could be otherwise in a more realistic spacetime model: for thermodynamic reasons one supposes that conditions in the contracting phase of a Bang/Crunch cosmology will be quite different to those in the expanding phase. In the euclidean version of the conformally related space, this could mean that there will be a sudden jump in the geometry as one passes through $\theta = \pm\pi$. This is exactly the situation for a euclidean version of an "asymmetric brane" spacetime — see for example [74]. Using these techniques one can try to account for the jump by [at least formally] assigning a stress-energy tensor to a "brane" at $\theta = \pm\pi$ in the conformally related space. It remains to be seen whether suitable asymmetric brane-like junction conditions can be formulated in such a way as to produce a more realistic cosmology.

The identification of the Crunch with the Bang is also an interesting move from the point of view of the "cyclic" cosmology [52]. It is not known definitely whether this cosmology "cycles" *eternally*; but if it does, then it is natural to ask whether the successive "lifetimes" of the cosmos are really distinct. A cyclic conformal time coordinate would of course be perfectly natural in such a case. This would be particularly interesting if it can be established, as suggested in [75], that string or M theory allows an orderly evolution through cosmological singularities.

To summarize briefly: AdS_4 cosmology leads naturally to an apparently disconnected boundary for the euclidean version of the spacetime. Holography seems to require that the two boundary components should be identified, an option which is always open in the cosmological case since the boundary components must have the same topology. In this section we have given a very simple example in which this identification is actually suggested by the physics of the matter content of the spacetime. It turns out that the identification has two surprising properties: it involves a non-trivial "twist" as the identification is performed, and it may possibly necessitate a similar identification in the lorentzian version. The main point is that, contrary to appearances, these spacetimes are *not* fundamentally incompatible with the holographic principle. The "double" boundary is merely a signal of unexpected complexity in the topology of the conformal compactification.

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5. Conclusion

The principal lessons of this investigation can be stated very briefly as follows.

First, there is no evidence, theoretical or [of course] observational, that the current cosmic acceleration is permanent. As soon as this is granted, however, one should begin to question whether the Universe is now or ever has been in a de Sitter-like state. An alternative is that it could be in an "anti-de Sitter-like" state.

Second, however, the result of adding matter to AdS is *not* a spacetime that resembles AdS: because of the singularity theorems, a cosmological AdS spacetime will typically have both a Bang and a Crunch. Thus the spacetimes considered by Maldacena and Maoz [23] are in this sense generic.

Third, the euclidean versions of the spacetimes so obtained will present the appearance of having a disconnected conformal infinity. As is so often the case in curved spacetimes, however, this is a situation in which coordinates can be very deceptive. The question as to whether infinity is disconnected *cannot* be settled by inspecting the form taken by the metric in some coordinate system.

Fourth, a disconnected infinity can be avoided — as holography suggests that it should be — by accepting that the conformal compactification may have a more complex topology than appears at first glance. The degree of complexity should be determined by the physical nature of the matter content of the spacetime.

Naturally, having clarified the basic way in which holography may work in these cosmologies, we have yet much to do. One major task is to determine the precise nature of the field theory at infinity which corresponds to the axion in the euclidean bulk. Another is to understand the role of the topology of the spatial sections of the Universe: as we have emphasised, this topology is of basic importance in cosmic holography whether or not it can be observed at the present time. In particular it is curious that *both* the spatial sections and the conformal compactification of the whole [euclidean] spacetime have the topology of compact flat manifolds. These spaces are known to have very remarkable geometric properties — for example [43], an initially arbitrary metric on them has to be *exactly* flat if its scalar curvature vanishes. We intend to discuss the physical consequences elsewhere.

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